Development And Change in the Social Fabric: A Statistical Mechanical Analysis Using Quantum Phase Transition and LMG Model

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ABSTRACT

We have analyzed here the changing pattern of social order of an agricultural community during the transition period, when an industrial unit is being built up at that locality for a certain period of time, on the basis of a statistical model. We have mapped here the situation onto the phase transition in the Lipkin-Meshkov-Glick (LMG) model induced by a quench, the quench time being identified with the target time for the completion of the project. To map the system as a spin model we consider each individual as a spin. We take that an individual having independent profession and those having professions dependent on others correspond to up and down spin states. During the transition when people having independent profession of cultivation are forced to take odd jobs and become dependent on others, we associate this with a spin flip which occurs during phase transition of the LMG model. Indeed when phase transition is induced by a quench, we have defect formation causing spins to be flipped and the density of defects related to the fraction of flipped spins depends on the quench time.

Keywords: development, social bonding, entanglement entropy.

It is a historical fact that the formation of industrial units in a locality causes an overall development in that area. However even in the beginning period of industrial era, in the 18th century many thinkers apprehended that this might lead to the formation of some maligned scars in the society. Rousseau [1] conceived that industry (civilization) exercises an essentially disruptive function by putting an end to a natural relationship, though he recognizes that it cannot be done away with. Marx [2, 3] argued that through the production mechanism a man is not merely alienated from nature but also he is alienated from his own nature. This argument obviously rescues man from the theological dilemma of the fall of man [4]. A lot of debate occurred in this context among philosophers, sociologists and economists. It is an undeniable fact that when an industrial unit is set up in a locality it brings along with it the development of the locality which primarily involves the creation of new employment channels for local people and subsequent rise in the level of income. It is our contention here to look into the pattern of social change during the period of the establishment of an industrial unit in a certain locality. We shall study this by mapping it onto a statistical mechanical model.

When the industrial unit is set up in an arid land so that the traditional way of life of the local people engaged in agricultural activity is not disturbed, local unemployed youths get absorbed in the process of the development of industry. However when the industrial unit is set up at the cost of agricultural land, the traditional way of life of the people engaged in agricultural activity is disturbed which causes an overall change in the social fabric. When the construction of the industrial unit is completed and it starts functioning, a new social order is set up in the locality. However in the transition period when the construction of the industrial unit goes on, the social fabric of the society gets changed gradually. The pattern of this change depends on how fast the construction of the industrial unit goes on which gives a measure of the flow of the investment during this period.

In a case study in West Bengal, India, it has been observed that when in a cluster of villages where most of the people are engaged in agricultural activity, an industrial unit is being set up in agricultural land, local people get involved mainly in three different ways:

1. Some youths get trained so that when the industry starts functioning they are employed as skilled laborers.

2. Some people get temporary jobs as laborers, small scale contractors, servants, sweepers, or security personnel in the construction site.

3. Some others of the locality seek new channels of income through small-scale business in the peripheral area.

During this transition period most of the people change their traditional employment pattern, which causes a change in the social fabric of the society. In fact this causes an alienation from the traditional way of life as they have to take a different pattern of job for livelihood at a sudden moment of time causing a disruption in the social bonding pattern.

In rural agricultural society the population can be mainly decided into two types so far as their earning pattern is concerned:

i) People who are engaged in independent profession of agriculture having their own farm land or are self-employed in small scale business.

ii) People who are engaged in professions such that they are dependent on others such as land laborers having no or very small amount of land of their own.

When an industrial unit is set up, this orientation of the pattern of livelihood gets changed. In fact land owners, particularly owners of small land, who lose their agricultural land, are forced to take odd jobs so that people having independent profession become dependent on the employers. In this case a sort of dehumanization occurs as they lose their independent status and have to take odd jobs. This disrupts the social fabric and the entire social bonding pattern gets changed. During the transition time we may call this as the formation of social defects. When after the transition period industrial unit starts functioning, a new social order is realized with a new pattern of social bonding.

In this note we want to quantify the dehumanization factor and study the gradual change in the pattern of social bonding during the transition period when the industrial unit is being set up in an agricultural land disrupting the traditional way of life. We shall formulate the problem in a statistical mechanical model of a phase transition induced by a quench.

To this end we consider the Lipkin-Meshkov-Glick (LMG) model which demonstrates the mechanism of a phase transition in a many body system and was first introduced in the context of nuclear physics [5]. A specific property of the LMG model [6-8] is that here each spin interacts with every other spin which is akin to the agricultural society where every individual has same sort of interaction with every other individual and thus knit a tight social bond. To map this system as a spin model we consider each individual as a spin. We take that an individual having independent profession and those having professions dependent on others correspond to up and down spin states. During the transition when people having independent profession of cultivation are forced to take odd jobs and become dependent on others, we associate this with a spin flip which occurs during phase transition of the LMG model. Indeed when phase transition is induced by a quench, we have defect formation causing spins to be flipped and the density of defects related to the fraction of flipped spins depends on the quench time.

It is noted that we are considering here the concept of quantum phase transition (QPT) to study the socio-economic problem. In this context we may mention that the relevant social problem deals with the interaction of each individual with other individuals rather than that of the overall society. In this extreme micro-domain of social structure it is expected that interplay between quantum mechanics and classical physics will take place. Indeed such an interplay between classical and quantum phenomena is observed in zero temperature QPT. Quantum criticality can be approached in two different ways. At $T=0$ the non-thermal control parameters such as the magnetic field λ approaches the critical value $\lambda \rightarrow \lambda_c$. Also we may view that as $T \rightarrow 0$ the magnetic field λ approaches the critical value $\lambda = \lambda_c$. Thus in quantum critical region both quantum and thermal fluctuations have definite role [9]. The typical time scale for the decay of the fluctuations is the correlation time which diverges near criticality. The cross over from quantum to classical behavior will occur when the correlation time exceeds $\beta = 1/k_B T$ in a quench induced quantum phase transition. This suggests that zero temperature phase transition induced by a quench can be mapped onto a classical finite temperature phase transition when the temperature effect is incorporated in the quench time with $T \sim 1/\tau$.

In the initial state the system represents a combination of people having the nature of both independent and dependent profession. This is mapped onto a system of up and down spins so that they represent an entangled state. The entanglement entropy can be viewed as to represent the quantitative measure of social bonding. The transition period denotes the quench time in quantum phase transition (QPT) of the LMG model [6-8]. During this transition time the social bonding pattern gradually changes as more and more people are being forced to take various odd jobs and this is reflected in the change of the entanglement entropy with time. Finally at the end of the transition time, when the industrial unit starts functioning, the social bonding associated with the initial agrarian society vanishes and a new social order emerges. This is viewed as the vanishing of entanglement entropy at the end of the transition when all the spins become polarized.

As mentioned earlier we have considered that in the initial state each individual interacts with every other individual forming a well-knit social bond. This is reflected in the Hamiltonian of the LMG model where each spin interacts with every other spin. The Hamiltonian for the isotropic LMG model is given by [5]

$$H = \frac{1}{N}\sum_{i<j}\left(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y\right) + \lambda\sum_i \sigma_i^z. \qquad (1)$$

This can be written in terms of the total spin system

$$H = \frac{2}{N}\left(\vec{S}^2 - S_z^2 - \frac{N}{2}\right) + 2\lambda S_z, \qquad (2)$$

with $S^{\alpha} = \frac{1}{2}\sum_i \sigma_i^{\alpha}$ and λ being the external magnetic field. This highly symmetric interaction pattern introduces the loss of the notion of geometry as there is no distance between the spins. Indeed this system with $\lambda = 0$ can be mapped onto the initial state of the community in a specific locality when a spin in the system is taken to represent an individual of the community. Now we consider time dependent external magnetic field in (1). In the present context $\lambda(t)$ is taken to measure the flow of investment at a particular time. When an industrial unit is being constructed with a certain target time τ for completion, investment is gradually tuned to increase with time and we can map it onto the time dependent magnetic field $\lambda(t)$. In fact we consider the choice

$$\lambda(t < 0) = -\frac{t}{\tau}, \qquad (3)$$

where we take the target time τ as the quench time. This implies that the investment flow at an instant of time t is inversely proportional to τ so that for short (large) target time investment flow is large (small). When the project is completed and starts functioning we have $\lambda = 1$.

It has been pointed out by Kibble [10] that in second order phase transition the critical slowing down implies that when the system is driven through the transition its evolution cannot be adiabatic near the critical point. The non-adiabatic evolution in the critical region produces defects such that the system becomes a mosaic of ordered domains whose finite size depends on the transition rate. Later on Zurek [11] suggested the dynamical mechanism based on the universality of critical slowing down and predicted that the size of the ordered domains scales with the transition time τ as τ^{χ} where χ is the critical exponent. In recent times the Kibble-Zurek mechanism has been incorporated in zero temperature QPT by several authors [12-17]. In QPT apart from the study of the formation of defects it is also important to consider how entangled various parts of the system are with each other. When a bipartite quantum system is in a pure state the measure of entanglement between two subsystems is given by Von Neumann entropy. When the bipartite system is in a mixed state the entanglement of formation given by concurrence has the property that it reduces to the Von Neumann entropy in a pure state [18]. It has been observed that the entanglement entropy of a block of L spins with the rest of the system at criticality follows a logarithmic scaling law in one dimensional spin systems [19]. The basic feature associated with this result is that at criticality the correlation length diverges and the system becomes conformal invariant. The conformal symmetry leads to the logarithmic scaling law of the entanglement entropy at criticality [20]. When the criticality is induced by a quench the scaling behavior changes and depends on the quench time [16].

It is to be mentioned that in LMG model as there is no notion of distance as in other spin model the conformal symmetry does not show up. However it has been observed that the LMG model also shows the sealing behavior similar to other one dimensional spin models [19]. Indeed it has been noted that if we introduce point splitting regularization in the Hamiltonian given by (2) the regularized Hamiltonian exhibits conformal symmetry at criticality and vanishes at the sharp point limit [17]. The formation of defects as well as the entanglement entropy of a block of L spins with the rest of the system in LMG model when QPT is induced by a quench has been studied in [17]. It has been observed that the number density of defects is given by

$$n = \frac{2\sqrt{2}+1}{8\pi\sqrt{\tau}}, \tag{4}$$

Where τ is the quench time. This shows that the number of defects scales like $1/\sqrt{\tau}$. However it is to be noted that the Kibble-Zurek correlation length characterizing the typical distance between defects cannot be introduced here though we can estimate the fraction of flipped spins after the quench. It is observed that in the present context, the fraction of flipped spins denotes the number of individuals who have been forced to take odd jobs and thus become dependent on the employer from their independent profession. This is the quantitative estimate of social defects formed during the transition denoting the dehumanization factor. In fig.1 we have plotted the variation of the density of defects (n) vs. the quench time (τ) which here represents the span of the transition period.

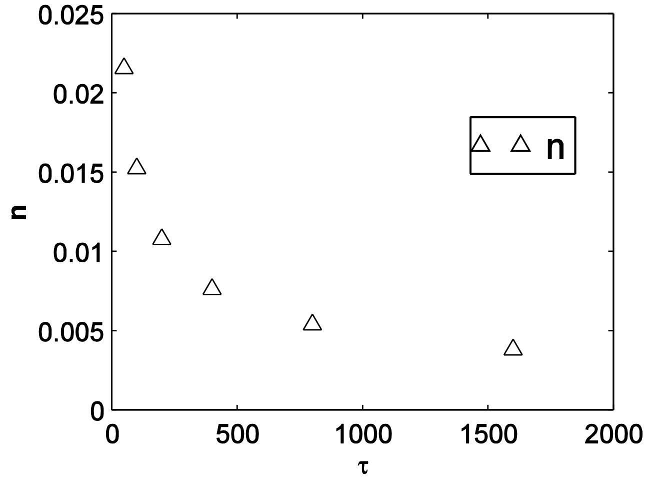

Fig.1: Variation of the number density of defects (n) with quench time τ .

The entanglement entropy $S(L,\tau)$ for a block of L spins with the rest of the system depends on the quench time and the size of the block L is restricted by a constraint. We have

$$S(L,\tau) = 0.37\frac{lnL}{ln\tau}, \quad (5)$$

with the maximum value of entropy S_{max} given by

$$S_{max} \approx 0.36 ln\tau + 1.85. \quad (6)$$

This puts a constraint on the block size L such that

$$lnL \leq 0.097(ln\tau)^2 + 0.5 ln\tau . \quad (7)$$

The system has two limit behaviors. At criticality $|\lambda|$ lies in the region $0 < |\lambda| < 1$. The entanglement entropy is maximum at $\lambda = 0$ and decreases with the increase in the magnetic field until at $\lambda = 1$ it vanishes when the system represents a product state. However within this region the scaling behavior remains the same. The change of the entanglement entropy with λ follows the relation

$$S_L(\lambda) = S_L(\lambda = 0) + \frac{1}{2}log_2(1 - \lambda^2). \quad (8)$$

It is to be mentioned that we cannot consider the notion of a block of L spins as a set of contiguous spins in LMG model and thus does not allow the study of any correlation between two spins or a block of spins with the rest of the system. This makes a great departure from the concept of entanglement entropy of a usual spin system with the LMG model. The mosaic of ordered domains here corresponds to the distribution pattern of the coherence number where all spins are aligned along the same direction intercepted by two excited spin states locating the position of defects (kinks).

When these results are transcribed to the present context of social change the coherence number is identified with the cluster size of people having more or less the same new job pattern or similar income distribution during the formative period and this depends on the square root of the target-time $\sqrt{\tau}$. This means that for large τ the cluster size of people having more or less same job pattern or income distribution is large. As the time rolls and the investment gradually increases different groups of people are engaged in different kind of jobs having various income distributions. Now we have noted above that as λ increases in the critical region $0 < |\lambda| < 1$, entanglement entropy gradually decreases until at $\lambda = 1$ it vanishes. This implies that with the gradual increase in investment during the formative period social bonding between certain groups of people in the initial set up with the remaining community decreases. In fact we can take that at the initial state ($\lambda = 0$), the entropy is maximum indicating that social bonding of a certain group of people with the remaining people in the community is maximum. As time rolls and investment flows the bonding gradually decreases as quantified in (8) until at the final state with $\lambda = 1$ when the industrial unit starts functioning the initial bonding vanishes when a new social order is set up. From (5) we note that the entanglement entropy decreases with the increase in quench time τ. In fig.2 we have plotted $S(L, \tau)$ showing the variation of S with τ for various values of L at $\lambda = 0$. In the present context this result suggests that when an investor with a fixed

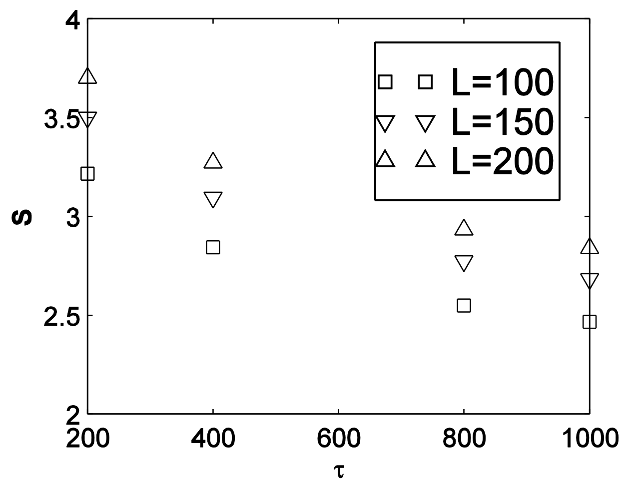

Fig.2: The variation of S with τ for different values of L, namely 100, 150 and 200.

amount of capital launches an industry (or several industrial units) with a specific target time (τ), the social bonding (S) for a certain number of people (L) in the initial state with the remaining community gradually decreases with the increase in target time. Indeed for large target time the coherence number which scales like $\sqrt{\tau}$ denoting the cluster of people having more or less the same new job pattern becomes large. This implies that for a number of people in the initial set up, the social bonding with the rest of the community decreases. It is noted that when the target time τ is very small number of defects will be large and the coherence number will be small so that there will be almost no correlation among the individuals. From (7) it is observed that in this case L will be very small so that the maximum size of the group of people in the initial set up having bonded with the rest of the community becomes negligible. This means that in this case the social order becomes almost randomized.

According to this analysis the number of defects scales like $\tau^{-1/2}$ indicating that for large (small) target time the dehumanization factor (social defects) is small (large). The pattern of social bonding changes such that as the time rolls and investment flows the bonding gradually changes. The bonding of a group of people in the initial set up with the rest of the community decreases as the target time becomes larger. In this analysis we give a generalized formulation and quantitative estimates of the changing pattern of the social order during the formation of an industrial unit in an agricultural locality based on a statistical mechanical analysis.

However there are some limitations and shortcomings of this particular manuscript also. No empirical data have been provided. We have just dealt with the conceptual aspect of the problem. We agree that empirical data would have been of much use here.

References:

[1] J.J. Rousseau (1762): Emile on de leducation (Garnier-Flammerion) Paris (1966).

[2] K. Marx (1867): Capital vol 1; Progress Publishers (1958).

[3] K. Marx (1894): Capital vol 3; Progress Publishers (1958).

[4] I. Meszaros: Marx's Theory of Alienation, Merlin Press (1970).

[5] H.J. Lipkin, N. Meshkov and A.J. Glick: Nucl.Phys. 62, 188 (1965).

[6] J. Khalouf-Rivera, J. Gamito, F. Perez-Bernal, J.M. Arias, P. Perez-Fernandez: Phys.Rev.E 107, 064134 (2023).

[7] Dong-Yan Lu, Guang-Hui Wang, Y. Zhou, L. Xu, Yong-Jin Hu, Wei-You Zeng, Qing-Lan Wang: Results in Physics, 21, 103832 (2021).

[8] Q. Wang, J. Khalouf-Rivera, F. Perez-Bernal: Phys.Rev.A 111, 032209 (2025).

[9] M. Vojta: Rep.Prog.Phys. 66, 2069 (2003).

[10] T.W.B. Kibble: J.Phys.A 9, 1387 (1976); Phys.Rep., 67, 183 (1980).

[11] W.H. Zurek: Nature 317, 505 (1985); Phys.Rep. 276, 477 (1996).

[12] W.H. Zurek, U. Dorner and P. Zoller: Phys.Rev.Lett. 95, 105701 (2005).

[13] J. Dziarmaga: Phys.Rev.Lett. 95, 245701 (2005).

[14] L. Cincio, J. Dziarmaga, M.M. Rams and W.H. Zurek: Phys.Rev.A 75, 052321 (2007).

[15] B. Basu and P. Bandyopadhyay: J.Phys.A, 43, 354023 (2010).

[16] P. Majumdar and P. Bandyopadhyay: Phys.Rev.A 81, 012311 (2010).

[17] B. Basu, P. Bandyopadhyay and P. Majumdar: Phys.Rev.A 83, 032312 (2011).

[18] T.J. Osborne and M.A. Nielsen: Phys.Rev.A 66, 032110 (2002).

[19] J.L. Latorre and A. Riera: J.Phys.A.Math.Theor., 42, 054002 (2009).

[20] C. Holzhey, F. Larsen and F. Wilczek: Nucl.Phys. B, 424, 443 (1994).

[21] B. Basu, P. Bandyopadhyay and P. Majumdar: Phys.Rev.A 86, 022303 (2012).

[22] B. Basu, P. Bandyopadhyay and P. Majumdar: Phys.Rev.A 92, 022343 (2015).